RESOLUTION OF COMPLEX DECAY CURVES BY ANALOG COMPUTER TECHNIQUES

DANA PECKWORTH

RESOLUTION OF COMPLEX DECAY CURVES

BY

ANALOG COMPUTER TECHNIQUES

by

Dana Peckworth

Lieutenant, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
CHEMISTRY

United States Naval Postgraduate School
Monterey, California

1959

RESOLUTION OF COMPLEX DECAY CURVES

BY

ANALOG COMPUTER TECHNIQUES

by

Dana Peckworth

This work is accepted as fulfilling

the thesis requirements for the degree of

MASTER OF SCIENCE

IN

CHEMISTRY

from the

United States Naval Postgraduate School

ABSTRACT

Resolution of complex exponential curves encountered in the study of beta-active substances can yield useful information about the component activities and their decay constants or absorption coefficients. The electronic analog computer is considered as a means of providing a more rapid resolution than can be obtained by the usual graphical techniques. A computer resolution method is described with its advantages and limitations. This method is applied to representative composite beta decay curves and a beta energy absorption curve.

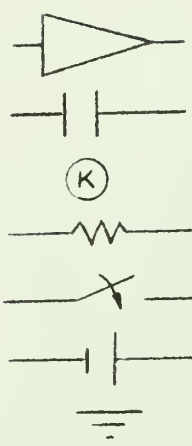
The writer wishes to thank Professor W. W. Hawes and Professor R. A. Reinhardt of the Department of Metallurgy and Chemistry for their willing advice and assistance.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

TABLE OF SYMBOLS

Symbols



- High-gain DC amplifier
- Capacitor
- Coefficient potentiometer
- Resistor
- Switch
- DC voltage source
- Ground

Letters

A,B,C, etc.	- Components in a complex decay scheme
C	- Capacitance
e	- Voltage
i	- Current
R	- Resistance
t	- Time
$t_{\frac{1}{2}}$	- Half-life

Subscripts

f	- Feedback
g	- Grid
i	- Input
o	- Output

Greek letters

$\alpha, \beta, \gamma, \varphi$	- Scaling factors
λ	- Decay constant
β^-	- Beta particle
τ	- Computer time
σ	- Thermal neutron cross section
φ	- Thermal neutron flux

CHAPTER I

INTRODUCTION

1. Background

When a mixture of independently decaying beta-active substances is counted, the activity is the sum of the several independent activities. The logarithm of activity plotted against time provides a curve convex toward the origin. The curvature results because the shorter-lived components are relatively less significant as time increases. After a sufficient time, the longest-lived component will predominate; and the half-life of that component can be determined from the slope of the latter portion of the curve. The decay curve for the longest-lived component (which is a straight line on semi-log paper) is then subtracted from the composite curve. A residual curve remains which can be further resolved into the remaining components.

The resolution of such curves encountered in the study of beta-active substances can yield useful half-life information. The graphical techniques usually employed in resolution of complex decay curves are only approximate, and the degree of approximation increases greatly with the number of components. R. A. Lauderdale and A. H. Emmons [1] employed this technique for resolution of a decay curve and an absorption curve in their study of decontamination of radio-active water. An example of a complex decay curve is shown in Fig. 1.

In principle, any composite curve of independent activities can be resolved by graphical means. However, experimental uncertainties make the task tedious or impossible in the case of systems containing more

than three components or even two components if their slopes differ by less than about a factor of five.

2. Discussion

The utility of an electronic analog computer in the study of chemical kinetics has been clearly shown [2]. The same differential equations applicable to radioactive decay appear in chemical kinetics and can be represented rather simply by electrical analogs. For this reason, it was felt that an electronic analog computer might be used successfully in the resolution of composite curves to overcome the shortcomings of graphical techniques.

This investigation was undertaken to determine whether resolution of curves of this type was feasible, and if feasible, to develop a simple and rapid technique of resolution employing an analog computer and the minimum associated equipment. Data from representative beta decay schemes could be used to test the validity of any computer techniques developed.

The generation of exponential curves by electrical analogs was accomplished by standard analog computer techniques. Normally, analog computers are used to study the effects of various parameters on the solution of differential equations. The values of the parameters are known beforehand, and the problem is one of programming the computer so that the effects of the parameters can be observed. In this investigation, the analog computer was used as a function generator. The equations involved were solved by the computer in order to generate the desired complex exponential curve. The parameters, in this case, were the unknowns which had to be adjusted until the generated curve fitted the observed data. A priori, an exponential decay curve with an unknown

number of components can be resolved by successive approximations. One of the important considerations in this study was a rapid means of comparing a computer generated decay curve with the actual data to be resolved.

Several analog computer techniques for resolution were tested using data obtained by conventional counting techniques. Radioactive samples were obtained by irradiating certain inorganic compounds in an AGN 201 reactor. Particular attention was given to the problem of resolving systems containing components whose half-lives differed by a factor of about five or less which were most difficult or impossible to resolve by graphical means. The applicability of computer resolution was further investigated for beta energy absorption studies in which the data can be closely represented by an exponential law over a limited range.

CHAPTER II

EQUIPMENT

1. The Electronic Analog Computer

An Electronic Analog Computer consists of a number of high-gain DC electronic feedback amplifiers with accessories which include initial voltage sources, resistors, capacitors, potentiometers and a problem-board arrangement for making connections. The computer solves problems by electrical analogy as its name implies. A program is devised for the computer so that the electrical equations are of the same mathematical form as the equations for the physical system. Time-dependent variables are represented by voltages; the amplifiers being used to perform the operation of summing several voltages or that of summing and integrating. Thus, with the proper use of scaling factors, solutions for differential equations may be obtained from a recorded output. After a problem has been set up on the computer, the effects of various parameters may be studied by merely adjusting initial value voltages, coefficient potentiometers or values of resistance and capacitance. To change the value of a parameter requires only seconds thus permitting a high degree of versatility in the solution of differential equations. The two basic amplifier connections and the electrical equations applicable are illustrated in Fig. 2.

Two Donner Model 30 Analog Computers with associated accessories manufactured by the Donner Scientific Company of Berkeley, California, were available for use in this investigation. The model 30 contained ten operational, high-gain DC amplifiers with an average gain over full

range of greater than 10^4. The output voltage was limited to $\pm$ 100 volts with load currents up to $\pm$ 5 ma. The amplifier short term drift was reported [3] to be less than 2 mv under normal conditions and less than 4 mv/hr over a long period. Five isolated, initial condition, power supplies were available with an output up to 100 volts with a 5 ma load. The computers had a "Hold-Operate" switch to allow interruption of the solution for short periods of time.

Associated equipment included Donner Problem Boards Model 30-6, Precision Potentiometer Boards Model 30 and precision plug-in computing elements. Resistance and capacitances were of the precision plug-in type accurate to $\pm$ 1%.

2. Recording devices

The output or solution provided by an analog computer may be observed in a number of ways: (1) by the reading of a high resistance voltmeter, (2) by a cathode ray oscilloscope trace, and (3) by various types of graphical or pen recorders.

Two types of graphical or pen recorders were used in this investigation. The first was the Sanborn 150 four channel brush recorder manufacturered by the Sanborn Company of Cambridge, Massachusetts. The Sanborn 150 recorder consisted of two dual channel DC amplifiers, model 150-2000 [4] each designed for measuring and recording two separate single-ended signals. Input sensitivities were available in increments of 1 to 100 volts/cm. Using both amplifiers, four signals could be recorded simultaneously at recording speeds in increments from 0.25 to 100 mm/sec. Maximum signal input was 250 volts for a full scale deflection from peak positive to peak negative of 500 volts. The minimum useable signal was 0.1 volt.

The second type was a Moseley Autograph Model 3 X-Y Recorder [5] manufactured by the F. I. Moseley Company of Pasadena, California. This recorder was a two axis, graphic recorder designed to record on a horizontal plane using 8½" x 11" graph paper. Recording was accomplished by a pen carried on a dual axis carriage. Each axis had voltage ranges in increments from 5 millivolts to 500 volts full scale and a "fixed-variable" selector so that the maximum voltage of each range could be extended by adjusting a potentiometer. The manufacturer claimed an accuracy of 0.25% of full scale on all ranges when delivered.

The Moseley Autograph was equipped with a curve follower adapter [6] which converted the recorder to a function generator. An output voltage proportional to a conducting curve was available when the machine was used as a function generator. A pickup stylus was designed to follow a conducting curve as the X-axis of the recorder was driven by any desired controlled or constantly varying input voltage. The output voltage was proportional to the reading of the curve on the Y-axis as a function of the input to the X-axis.

3. Comparator bridge

The Donner Comparator Bridge Model 50 [7], manufactured by the Donner Scientific Company of Berkeley, California, was an instrument designed as a precision voltage comparator and a precision voltage source. The internal voltage standard was set to 100 volts ±0.1% with a long term accuracy of 0.5%. Settings of coefficient potentiometers with resistances from 10K ohms to 200K ohms could be read to 0.1% accuracy by a direct reading dial. Higher resistance potentiometers could be measured to 1% accuracy. The Comparator Bridge could be used to measure DC voltages up to 100 volts when used as a null detector.

4. Radiation particle counting equipment

The apparatus used for counting beta activity consisted of a Geiger-Mueller counter with shield and a binary scaling unit.

The Geiger-Mueller tube, Model TGC-2, manufactured by the Tracerlab, Inc. of Boston, Massachusetts, was a mica end-window type with window thickness of 1.9 mg/cm^2 and diameter of 1.1 inches. The Geiger-Mueller tube was helium filled and organic vapor quenched. Dead time was 10^{-4} seconds.[1]

The Scaling Unit, Model 1070A was manufactured by the Atomic Instrument Company of Cambridge, Massachusetts. It was a binary scaler including its own power supply, timing device and automatic reset. The main characteristics were 8 resolving time, 1 microsecond; stop clock accuracy, 0.02 minutes; timer accuracy, 1 second. A detailed description of this apparatus was given by Morey [9].

5. Research reactor

The radioactive samples counting in this investigation were irradiated in the AGN 201 reactor at the United States Naval Postgraduate School, Monterey, California. The AGN 201 is a small research reactor with a thermal neutron flux of approximately 4.5×10^6 neutrons/cm^2 sec.

[1] Tube data was provided by the manufacturer.

CHAPTER III

PROCEDURE

1. Programming the Analog Computer to generate a complex decay curve

In terms of activity, the equation for exponential decay is usually written:

$$A = A_o e^{-\lambda t} \tag{1}$$

Where A = measured activity at any time, t

A_o = initial activity

Equation (1) also may be expressed in the differential form:

$$dA = -\lambda A dt$$

Integrating

$$A - A_o = -\int_o^t \lambda A dt$$

Expressed in suitable terms to be solved by electrical analog, equation (1) finally becomes:

$$A = -\int_o^t \lambda A dt + A_o \tag{2}$$

It is general practice to use the operator, P, which is defined:

$$P \equiv \frac{d}{dt}$$

The operator is used to denote integration as follows:

$$\frac{1}{P}[f(x)] \equiv \int_o^t f(x) dx$$

Utilizing this notation, equation (2) may be expressed:

$$A = -\frac{1}{P}[\lambda A] + A_o \tag{3}$$

In a mixture of radioactive species with independent activities, the observed total activity is the sum of the component activities. Associated with each activity is a detection coefficient which is in general different for each activity. Since the complex curves to be resolved will be on the basis of relative activity, the detection coefficients will not affect the equations to be solved, but must be considered in converting the results into terms of absolute activity if so desired. Of primary interest is the slope of each component which is unaffected by the detection coefficient. The discussion of programming which follows assumed a complex decay curve composed of two independently decaying components. The equations used to describe the decay of each component were:

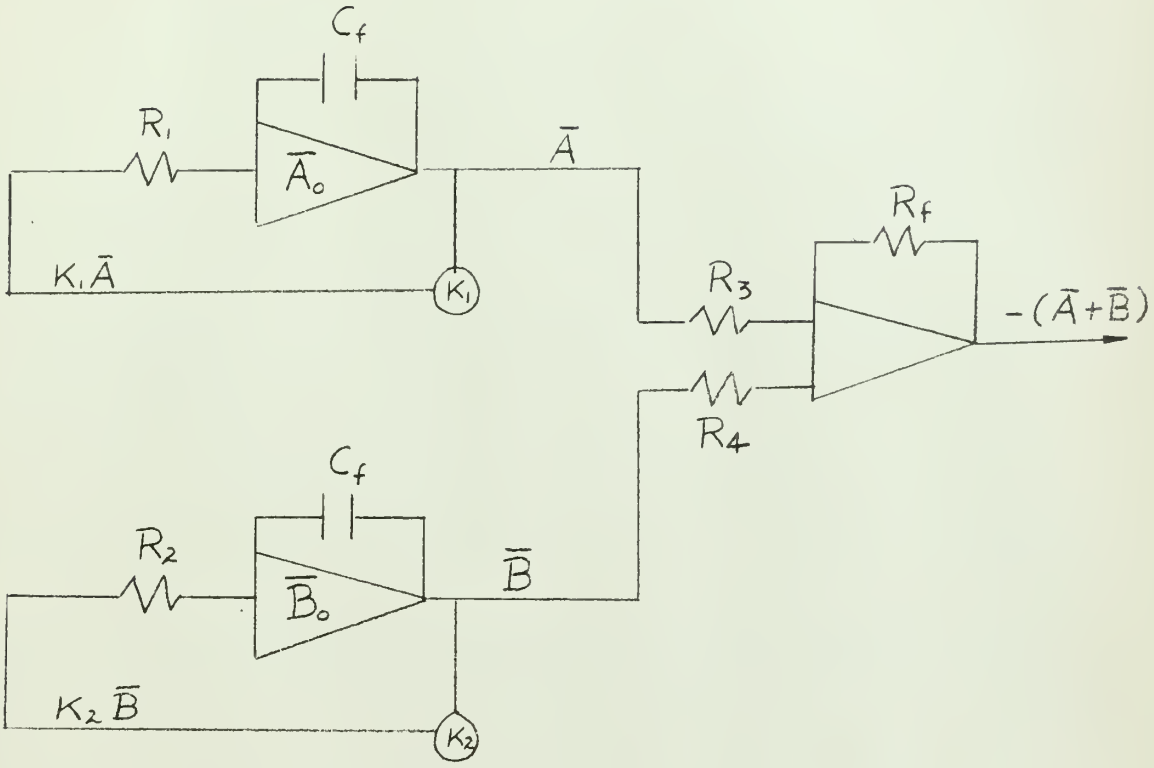
$$A = -\frac{1}{P}[\lambda_1 A] + A_o \qquad (4)$$

$$B = -\frac{1}{P}[\lambda_2 B] + B_o \qquad (5)$$

The sum of equations (4) and (5) gives the total activity of both components present at any instant. At zero time, the total activity is equal to the sum of the initial activities.

$$\text{Total Activity} = A_o + B_o \qquad (6)$$

To solve the equations involved, two integrators and a summing amplifier were used.



The bar on the symbols shown (i.e., $\bar{A}$, $\bar{B}$, etc.) indicated computer values which differed from actual value by scaling factors.

2. Scaling

By the design of the computer, voltages used were limited to ± 100 volts. Therefore, the problem was scaled to remain within these limits. Methods of scaling are discussed at length by Wheeler [10] and with a slightly different approach by Korn and Korn [11]. In this investigation, scaling was accomplished in the following manner.

In general, activity is measured in units which are too large to be scaled to the computer directly. The actual activity must be divided by

some factor α to reduce the activity to a value which can be represented by a voltage within the limitations of the computer.

$$\text{Scaled Activity, } a = \frac{\text{Actual Activity}}{\alpha}$$

Thus, α is not related to the computer scaling factors and must be considered separately when converting computer output voltages and initial value voltages to activity values.

The scaled activity was substituted in equation (2) which gave the equation to be solved.

$$a = -\frac{1}{p}[\lambda_1 a] + a_0 \tag{7}$$

The equation of the integrating amplifier used to solve equation (7) was:

$$\bar{A} = -\frac{1}{p}\left[\frac{K_1 \bar{A}}{R_1 C_f}\right] + \bar{A}_0 \tag{8}$$

In order for the analogy to be correct, equations (7) and (8) must be equal and

$$\lambda_1 a t = \frac{K_1 \bar{A} \tau}{R_1 C_f} \tag{9}$$

from which

$$R_1 C_f = \left(\frac{K_1}{\lambda_1}\right)\left(\frac{\bar{A}}{a}\right)\left(\frac{\tau}{t}\right) \tag{10}$$

Computer scaling factors were defined as follows:

	Computer Values		Actual Values
1. time	τ	=	βt
2. slope	K	=	$\phi \lambda$
3. activity	$\bar{A}$	=	γa

Substituting the scaling relationships into equation (10), the following expression was obtained:

$$R_1 C_f = \phi_1 \gamma_1 \beta \qquad (11)$$

The above expression must be satisfied using appropriate values of R_1 and C_f. Resistances were limited to values from 100K ohms to 10 megaohms and capacitances to .1 microfarad and 1 microfarad.

As a general rule, it was satisfactory to allow the scaled activity for each component to equal the computer voltage. In cases where the voltage necessary to represent a component was in the order of one to five volts, it was necessary to introduce the scaling factor γ in the integrating amplifier and eliminate it in the subsequent summing amplifier. Eliminating the scaling factor γ was necessary because the output voltage from the summing amplifier must represent each component in its true relationship.

In the summing integrator each input may be attenuated by the factor $\frac{R_f}{R_i}$. This factory may be used to eliminate the scaling factor γ which might have been introduced to scale up the input to one of the integrating amplifiers. However, in most cases the summing amplifier was used only to sum the outputs from the integrating amplifiers and, therefore, in each case R_i equalled R_f. An example of scaling activity data to an analog computer will be discussed later.

3. Computer solution recorded with the Sanborn 150 brush recorder.

The Sanborn 150 brush recorder was used as the recording device at the beginning of this investigation. In order to provide a rapid means of comparing the data to be resolved with the computer solution, a transparent overlay was used. The overlay consisted of a graph of the data plotted to a time and voltage scale compatible with the scale being used to record the solution. After each adjustment of computer parameters, the computer solution was recorded and compared to the overlay curve. The computer parameters were adjusted until the solution fitted the overlay curve.

4. Use of a function generator

The second method of resolving complex decay curves involved using the Moseley Autograph X-Y Recorder as a function generator. The data to be resolved was plotted to a suitable scale on orthogonal, 20 x 20 to the inch graph paper. A conducting curve was prepared by painting over the original curve with silver conducting paint according to the directions set forth in the Autograph Curve Follower instruction manual. A time base to drive the X-axis of the recorder was obtained from the circuit shown in Fig. 3. The output voltage of the function generator and the time base circuit were scaled to conform to the computer scale. Suitable switching arrangements were devised so that the computer and function generator outputs commenced at the same time. Their respective outputs were compared in a summing amplifier and the difference recorded on the Sanborn 150 brush recorder. The schematic circuit diagram showing the employment of a function generator is given in Fig. 4.

5. X-Y Recorder used in resolution of complex decay curves

Fig. 5 is a schematic circuit diagram showing how the Moseley Autograph X-Y Recorder was employed to record a computer solution. One of the primary considerations in the investigation of recording devices was a rapid means of comparing the computer generated exponential curve with the actual data. This was accomplished with the X-Y Recorder by plotting the data on orthogonal coordinates to a suitable scale compatible with the recorder and computer. The computer was programmed to generate a curve to the same scale and allowed to drive the Y-axis of the recorder. The X-axis was again driven by an input from the time base circuit previously described. With the curve representing the actual data on the recorder, the computer solution was recorded on the same graph. A visual as well as a recorded indication of how the computer generated curve differed from the actual curve was obtained for each adjustment of parameters. When the computer parameters had been adjusted so that the two curves coincided as nearly as possible, the parameters were those of the desired solution.

6. "Hold Relay" feature of the computer employed to refine the computer solution

A further refinement to the computer solution was obtained by using the X-Y Recorder, and a comparison curve made use of the computer "Hold Relay" feature. Fig. 6 shows schematically the arrangement used. With the "Hold Relay" the computer solution could be interrupted at any point during the process of solving a problem. Employing this feature, the solution was stopped at the desired intervals so that the output voltage could be read with the Donner Comparator Bridge. A timer was connected into the computer in such a way as to start when the "Compute"

switch was actuated and to stop when the "Hold" switch was actuated. Thus, readings of the output voltage and time were obtained and compared to the original data. The computer solution was adjusted as necessary to obtain a better fit with the actual data.

7. Obtaining representative complex decay data

To obtain representative data with which to test resolution techniques, various inorganic compounds were irradiated in the AGN 201 research reactor. Compounds were chosen with the thought in mind of beginning with systems containing two components with slopes reasonably close together. Availability of reactor time precluded using isotopes with half-lives greater than about 30 hours due to the exposure time required to obtain sufficient activity. The activity after the (n, γ) reactions have taken place can be predicted from the expression:

$$A = N\sigma\Phi(1 - e^{-\lambda t}) \qquad (1)$$

where A = activity in disintegrations/ second

N = number of atoms of the isotope present

σ = thermal neutron cross section

Φ = neutron flux

λ = decay constant

In choosing suitable compounds, the relative amount of activity expected and the length of time required to obtain sufficient activity for efficient counting was of particular interest.

Three compounds were considered in the search for a compound which

[1]All half-life figures used in this investigation are from the "Table of Isotopes". [12]

would give a representative two component beta decay scheme after irradiation; $KMnO_4$, $Na_2HAsO_4 \cdot 7H_2O$ and MnI_2. The exponential decay curve for activated $KMnO_4$ will represent the beta activity of K^{42} ($t_{\frac{1}{2}}$ = 12.52 hours)[1] and Mn^{56} ($t_{\frac{1}{2}}$ = 2.576 hours). K^{41} from which K^{42} is formed by an (n,γ) reaction represents only 6.8%[1] of the potassium present in $KMnO_4$; the remainder being K^{39} which will form K^{40} ($t_{\frac{1}{2}}$ = 1.25 X 10^9 years) to such a small extent as not to affect the total activity noticeably. The semi-logarithmic plot of the activity obtained from an 88.9 milligram sample irradiated for 3 hours and 15 minutes was a straight line as closely as the data could be interpreted. The slope corresponded to a half-life of 2.64 hours, approximately that expected for Mn^{56}. The component half-lives differ sufficiently to cause noticeable curvature in the decay curve had there been enough activity from each component. It was concluded that K^{41} with its small natural abundance and cross section (1.1 barns) did not provide sufficient activity to be distinguished from that due to Mn^{56}.

A 300 milligram sample of $Na_2HAsO_4 \cdot 7H_2O$ irradiated for 13 hours in a thermal neutron flux provided the decay data shown graphically in Fig. 7. The barely discernible curvature was inadequate to permit resolution of the data by graphical or computer means. The lack of curvature was attributed to two factors (1) the half-lives of As^{76} ($t_{\frac{1}{2}}$ = 26.4 hours) and Na^{24} ($t_{\frac{1}{2}}$ = 14.97 hours) differ by a factor of only 1.77 and (2) Na^{23} (σ = .53 barns) has a small cross section in comparison to As^{75} (σ = 4.3 barns).

MnI_2 did not exhibit the shortcomings encountered in the previously

[1]Values for percent abundance and thermal neutron cross section are from the CHART OF THE NUCLIDES. [13]

mentioned samples. The (n,γ) reaction of the compound provided a two component decay scheme representing I^{128} ($t_{\frac{1}{2}}$ = 24.99 minutes) and Mn^{56} ($t_{\frac{1}{2}}$ = 2.576 hours); and the resulting complex exponential curve (Fig. 8) had ample curvature. Two samples weighing 20.6 milligrams and 13.1 milligrams respectively were exposed to a thermal neutron flux for 5 hours. The compound as prepared was deliquescent and therefore, was mounted on a polyethylene planchet and sprayed with clear lacquer to protect the sample during irradiation and counting.

One compound, CuI, was considered for a suitable three component system. Present in CuI are the two naturally occurring isotopes of copper, Cu^{63} (69%) and Cu^{65} (31%) with cross sections of 4.4 and 2.2 barns respectively. After irradiation, the beta activity will consist of contributions from Cu^{64} ($t_{\frac{1}{2}}$ = 12.80 hours), Cu^{66} ($t_{\frac{1}{2}}$ = 5.10 minutes) and I^{128} ($t_{\frac{1}{2}}$ = 24.99 minutes). Irradiation of an 80 milligram sample provided the data shown in Fig. 9. The curve is essentially linear above 100 counts/second (normalized) but has considerable curvature beyond. It had been expected that the rapidly decaying Cu^{66} would provide sufficient activity to influence the curvature initially. Due the low percentage and small cross section of Cu^{65} combined with the rapid decay which took place between irradiation and counting, the activity from Cu^{66} could not be resolved.

CHAPTER IV

DISCUSSION AND RESULTS

1. Discussion of the resolution techniques investigated

As implied in the introduction, of major interest in this investigation was the problem of rapid comparison of a generated curve with the data to be resolved. Programming the computer to generate a complex exponential curve is a relatively simple matter requiring little time. Even though the number of independent components present need not be known, to reduce the complexity of the problem, a prior knowledge of the magnitude of the parameters was assumed. Each component required one integrating amplifier and had two variable parameters associated with it; the decay constant and the initial activity. Since the decay constant was represented by the value of a coefficient potentiometer and the initial activity by an initial value voltage, a two component system had four independently variable parameters, a three component system had six and so on. It becomes necessary to find a comparison method which will readily show the effect of each parameter on the computer output.

The Sanborn 150 recorder with the width of recorded trace limited to 10 centimeters did not provide sufficient resolution. Furthermore, the accuracy obtained at below 5 volts input was poor. This recorder was not used other than to test its application to some rather easily resolved synthetic complex decay curves.

It may be supposed that the use of a function generator would offer a means of providing a satisfactory resolution technique. The problem is to program a function generator to yield a voltage proportional to

the data to be resolved. The computer solution and function generator output may be compared in a summing network and the difference observed in some suitable fashion. Resolution is a matter of adjusting the computer parameters until no difference is observed. Investigation showed that there were many difficulties involved in preparing a satisfactory conducting curve and accurately calibrating the recorder time base input with the computer time scale. Furthermore, the difference obtained between the computer output and the function generator output was small, so that the effect of changing the value of a computer parameter was not readily apparent. The function generator then is not satisfactory for resolution of decay curves.

A technique which yielded satisfactory results employed the Moseley Autograph X-Y recorder. It had the advantage of providing a visual indication representative of the difference between the computer-generated curve and the actual curve. The effect of each parameter could be observed rather easily. This technique had several inherent inaccuracies which were overcome by use of the computer "Hold Relay" feature. Some inaccuracy was introduced in plotting the actual data to be used as the basis of comparison on the X-Y recorder. Intercalibration caused other errors. These errors were eliminated after a reasonably good fit had been obtained between the computer output and the comparison curve by interrupting the computer solution with the "Hold" switch at suitable time intervals. The Comparator Bridge (or an accurate, high resistance voltmeter) was used to measure the output voltage. After each reading, the computer solution was begun at time zero to eliminate any error that might be caused by condenser leakage during the time the reading was

being taken. The parameters were adjusted as necessary to give the best possible fit with the actual data.

2. Computer resolution techniques applied to representative data

The resolution technique just mentioned employing the X-Y recorder was first applied to synthetic data containing no experimental uncertainties. The technique provided satisfactory resolution of two and three component systems with half-lives which differed by a factor of about four or greater. The initial values of activity for the smallest and largest components could not differ by more than 100. At greater values, the computer could not represent the decay of the smallest component.

Data representative of systems containing experimental uncertainties was obtained by the methods previously described. Two sets of data were irresolvable for the reasons already mentioned. These systems point up the fact that several interrelated conditions must be satisfied before a composite exponential curve can be satisfactorily resolved. Component slopes must differ by a factor of about 4 to 5 or greater and there must be sufficient activity from each component. Irradiated MnI_2 samples met the above requirements and provided data suitable for resolution. Fig. 10a shows the data plotted on orthogonal coordinates suitable for use as a comparison curve. The data scaled to the computer was used for convenience. The computer scaling factors and the resolved data obtained are tabulated in Fig. 10b; and the computer program is shown in Fig. 10c.

The CuI sample failed to yield a three component system but was satisfactorily resolved by using the remaining data after the activity had dropped below 100 counts/second (normalized). The data was treated

as a two component system and the activities extrapolated back to time zero. The sum of the initial activities determined in this fashion was subtracted from the original data. There was a residue activity representative of the short-lived copper isotope, but the data were not accurate enough to give any half-life information for that component. The values obtained by computer resolution were: Cu^{64}, slope = .0541 hours^{-1} corresponding to half-life of 12.8 hours and I^{128}, slope = 1.666 hours^{-1} corresponding to $t_{\frac{1}{2}}$ = 25.0 minutes. These values as well as those determined from the MnI_2 data agree closely with the literature values.

Complex exponential curves are not only encountered in studying beta active substances but also in beta energy absorption studies. The data can be approximated over a limited range by the same differential equations describing exponential decay. To show the applicability of computer resolution techniques to this type of data, an absorption curve representing Zr(.89 Mev β^-), Zr(.39 Mev β^-) and Nb(.16 Mev β^-) [14] was considered. This is a three component system in which two of the slopes differ by a factor of four. The absorption curve and the computer resolved components are shown in Fig. 11.

CHAPTER V

CONCLUSIONS

Resolution of complex decay curves by analog computer techniques has been investigated and shown to be feasible. A suitable technique was developed and tested on a limited amount of data. With the method described, computer resolution was satisfactory in two and three component systems in which the slopes differed by at least a factor of four and the initial values of the largest and smallest components did not differ by more than one hundred to one. Computer resolution was comparable in accuracy to that expected from graphical resolution. Computer resolution offered the greatest saving of time in cases in which approximate values of the parameters were known. This would be the case in resolving sets of data pertaining to the same system. After a successful program has been developed and the initial resolution completed, a considerable saving of time would be offered by a computer resolution of subsequent curves.

Composite curves in which one component contributes very little to the total activity is illustrated by the $KMnO_4$ data; and effect on the curve of too little activity of one component combined with a small difference in slopes is shown by the Na_2HAsO_4 data. Neither curve has the curvature needed for satisfactory resolution. Thermal neutron irradiated samples of MnI_2 and CuI provided beta decay schemes suitable for counting by conventional techniques and rapid resolution by an analog computer technique. The half-life information obtained agreed closely with current literature values.

The application of computer resolution to beta energy absorption data over the limited range adequately represented by an exponential law was shown with a three component, Zr-Nb absorption curve.

FIGURE 1

COMPLEX DECAY CURVE FOR A

BETA- ACTIVE SAMPLE

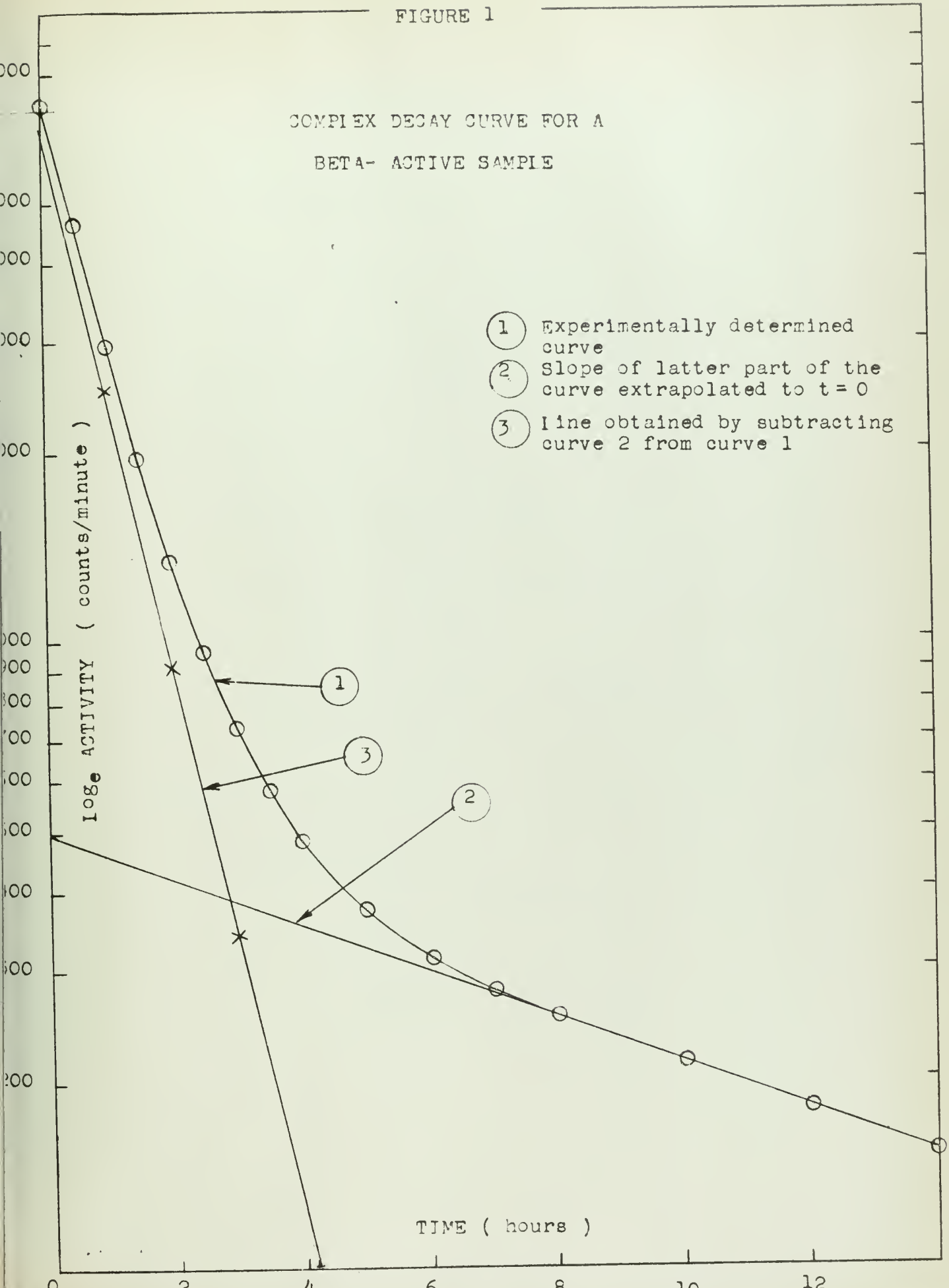
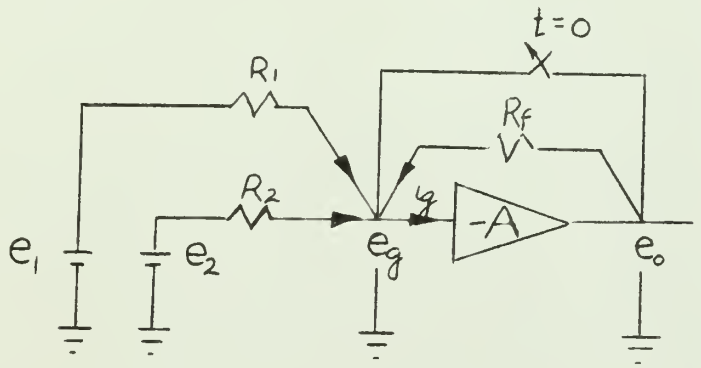


FIGURE 2

BASIC AMPLIFIER CONNECTIONS

Summing Amplifier



$$\frac{e_1 - e_g}{R_1} + \frac{e_2 - e_g}{R_2} + \frac{e_o - e_g}{R_f} = i_g$$

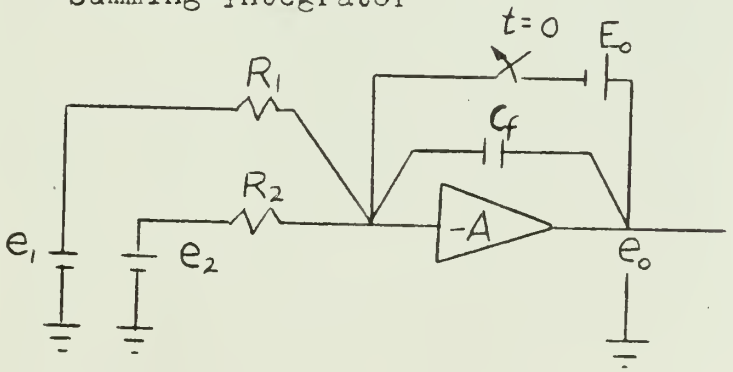
$|A| \approx 10^4$ AND $|e_g| \approx 10^{-4}|e_o|$

$\therefore \; e_g = i_g \cong 0$

Equation of a Summing Amplifier

$$e_o = -R_f \left(\frac{e_1}{R_1} + \frac{e_2}{R_2} \right)$$

Summing Integrator

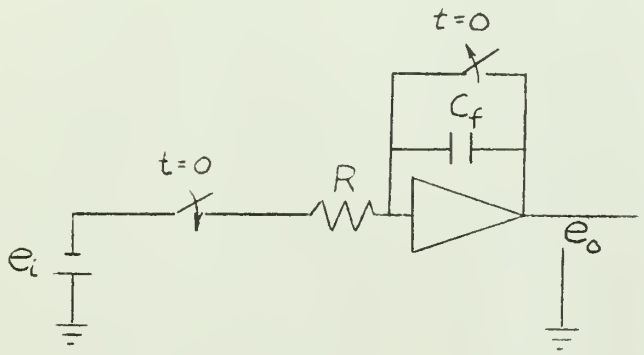


Equation of a Summing Integrator

$$e_o = -\frac{1}{p}\left[\frac{e_1}{R_1 C_f} + \frac{e_2}{R_2 C_f} \right] + E_o$$

FIGURE 3

GENERATION OF A TIME BASE



$$e_o = -\int_0^t -\frac{e_i}{RC_f}\,dt = \frac{e_i}{RC_f}\,t$$

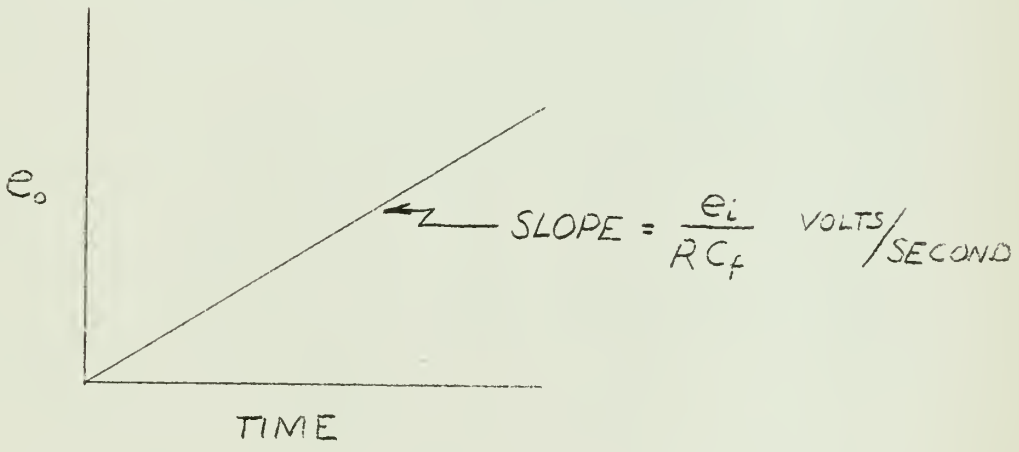


FIGURE 4

EMPLOYMENT OF A FUNCTION GENERATOR

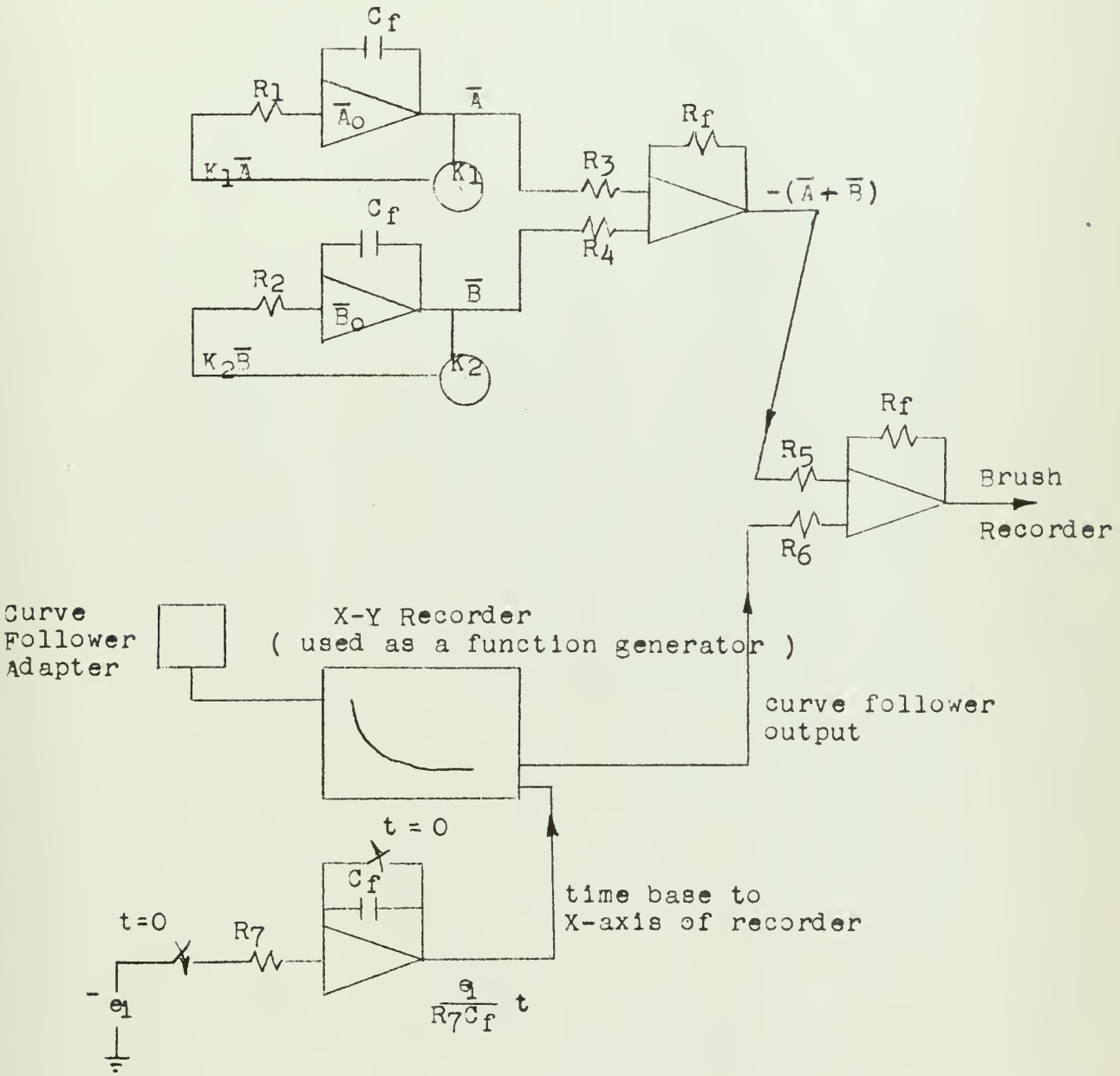


FIGURE 5

EMPLOYMENT OF AN X-Y RECORDER

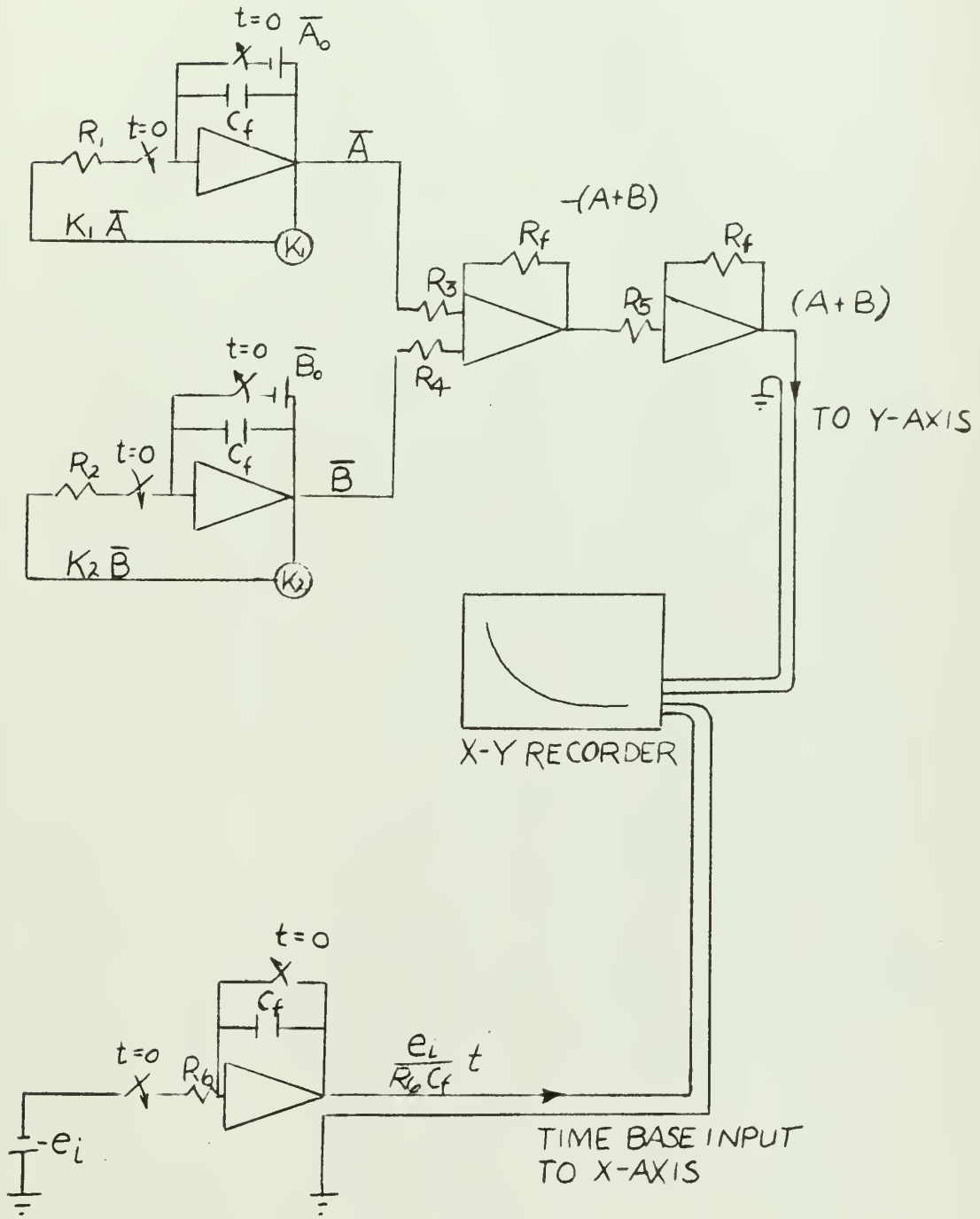


FIGURE 6

SWITCHING ARRANGEMENT TO

UTILIZE THE COMPUTER "HOLD RELAY"

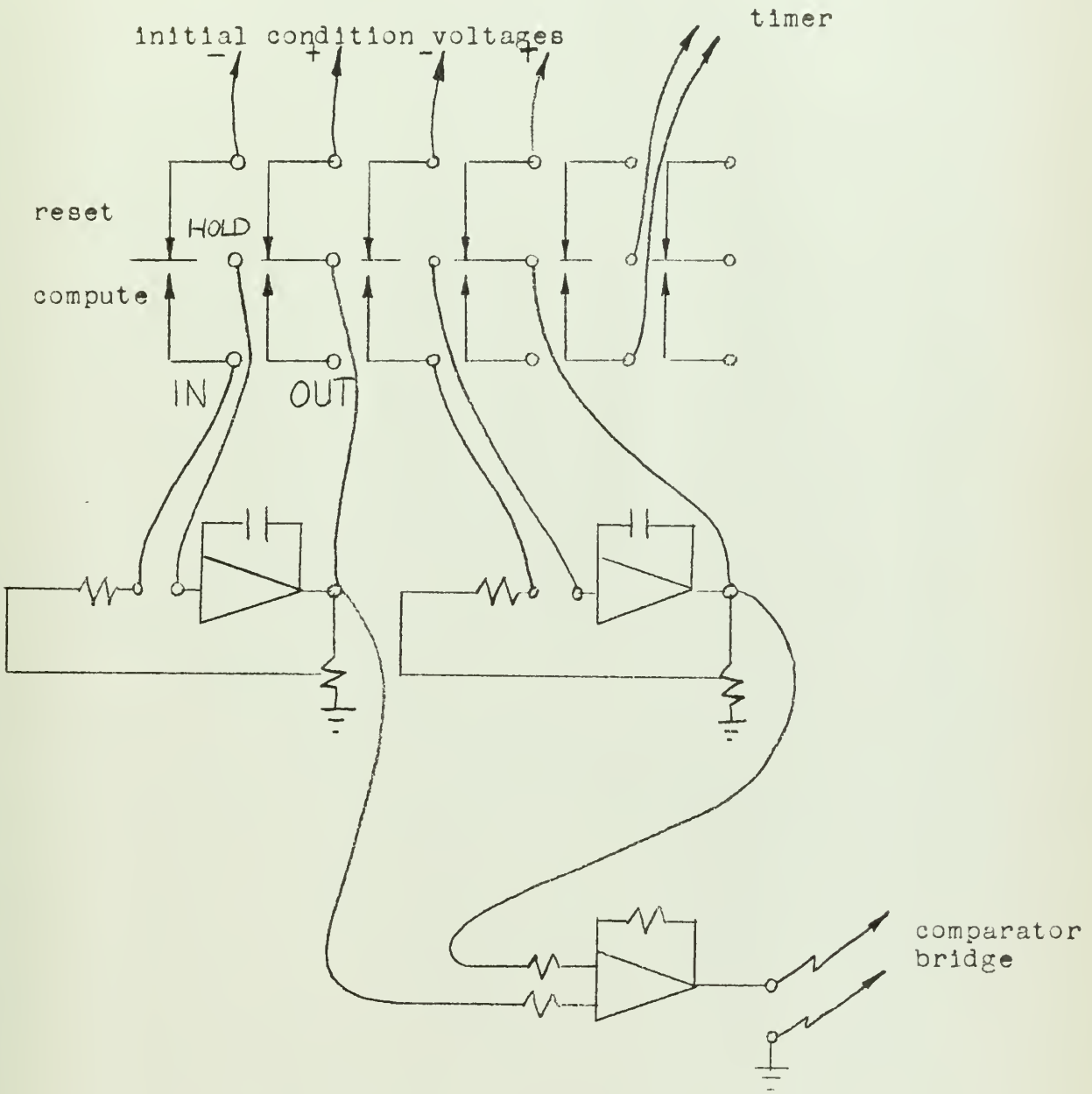


FIGURE 7

COMPLEX DECAY CURVE FOR

IRRADIATED $Na_2HAsO_4 \cdot 7H_2O$

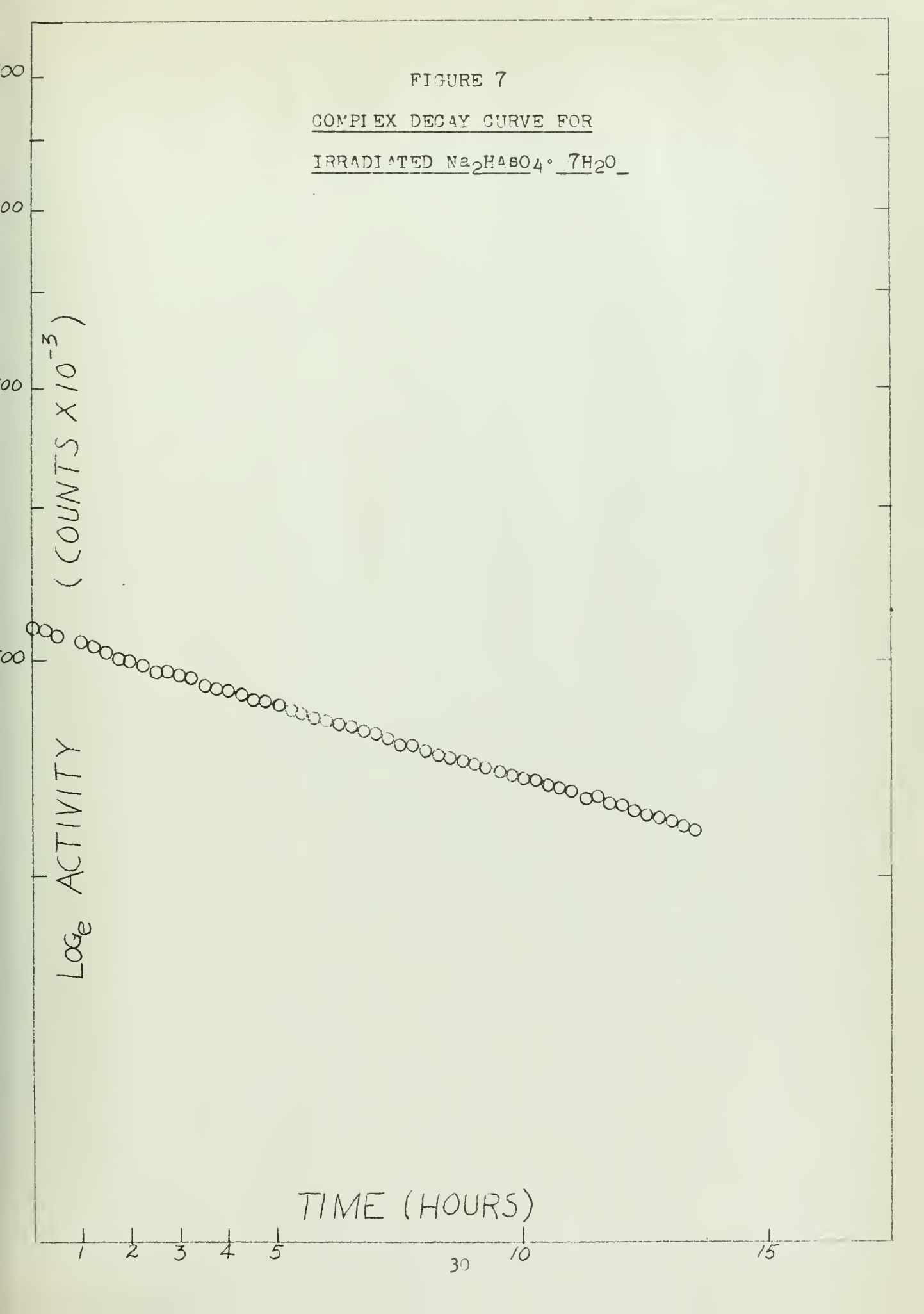


FIGURE 8

COMPLEX DECAY CURVE

FOR IRRADIATED MnI_2

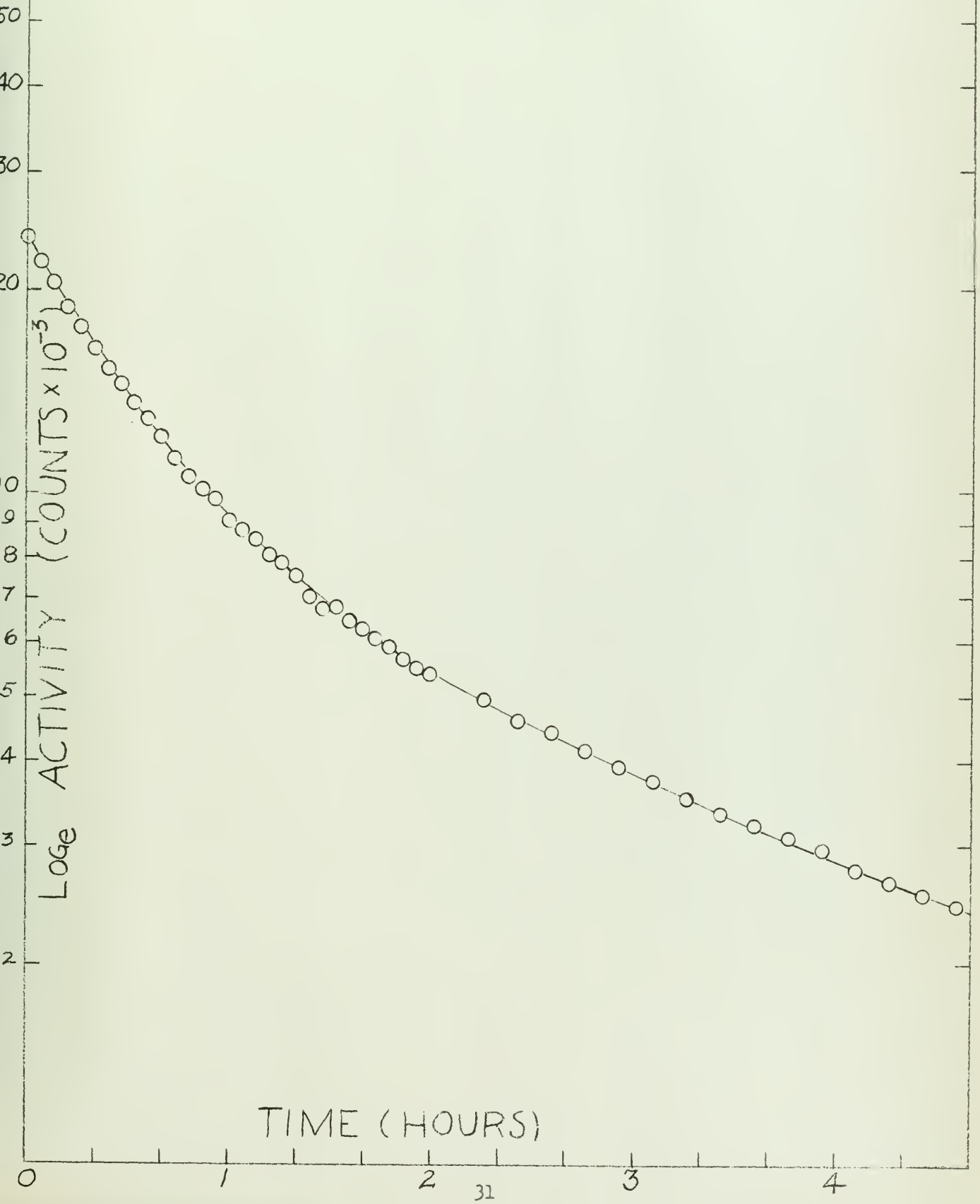
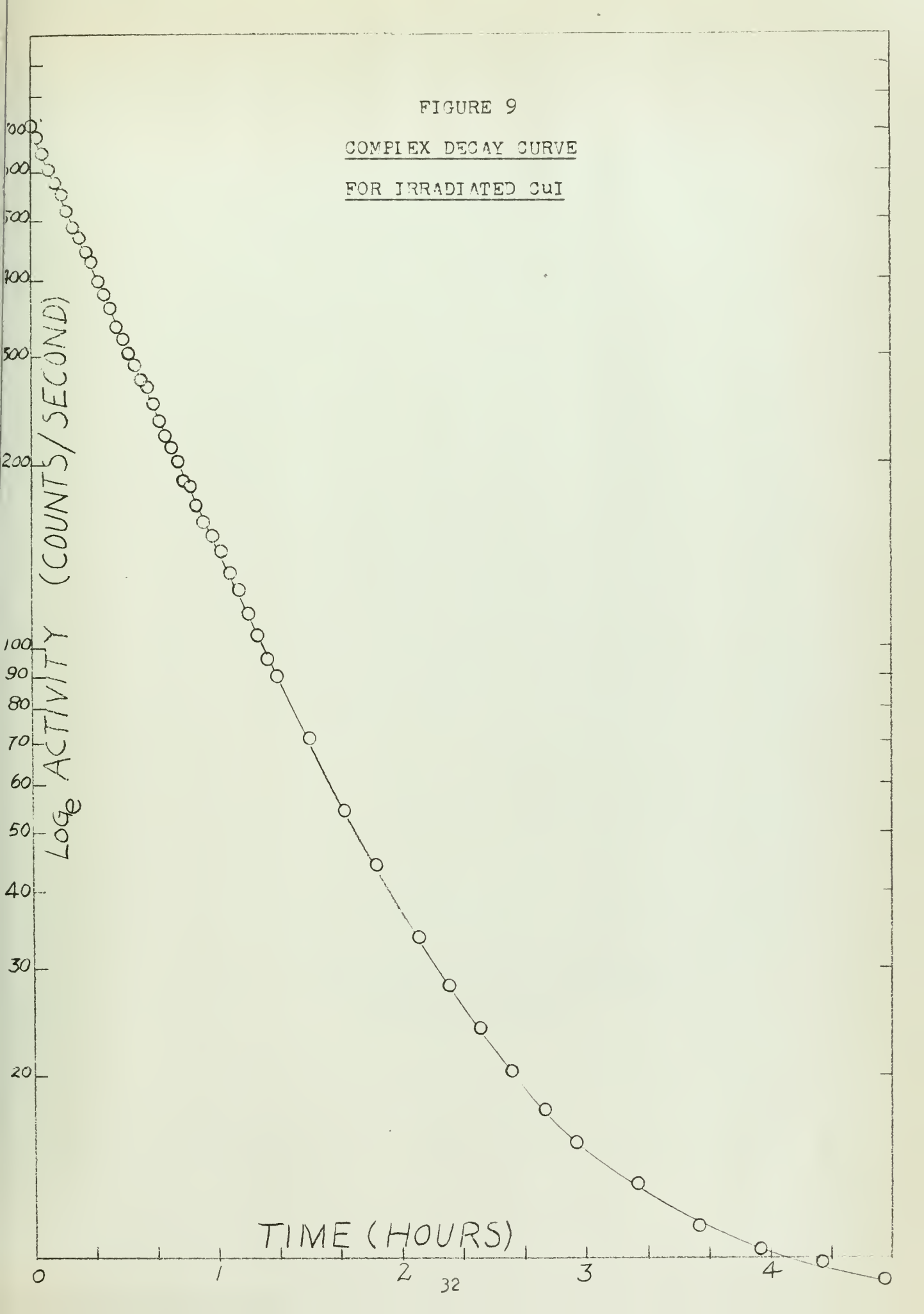


FIGURE 9

COMPLEX DECAY CURVE

FOR IRRADIATED CuI



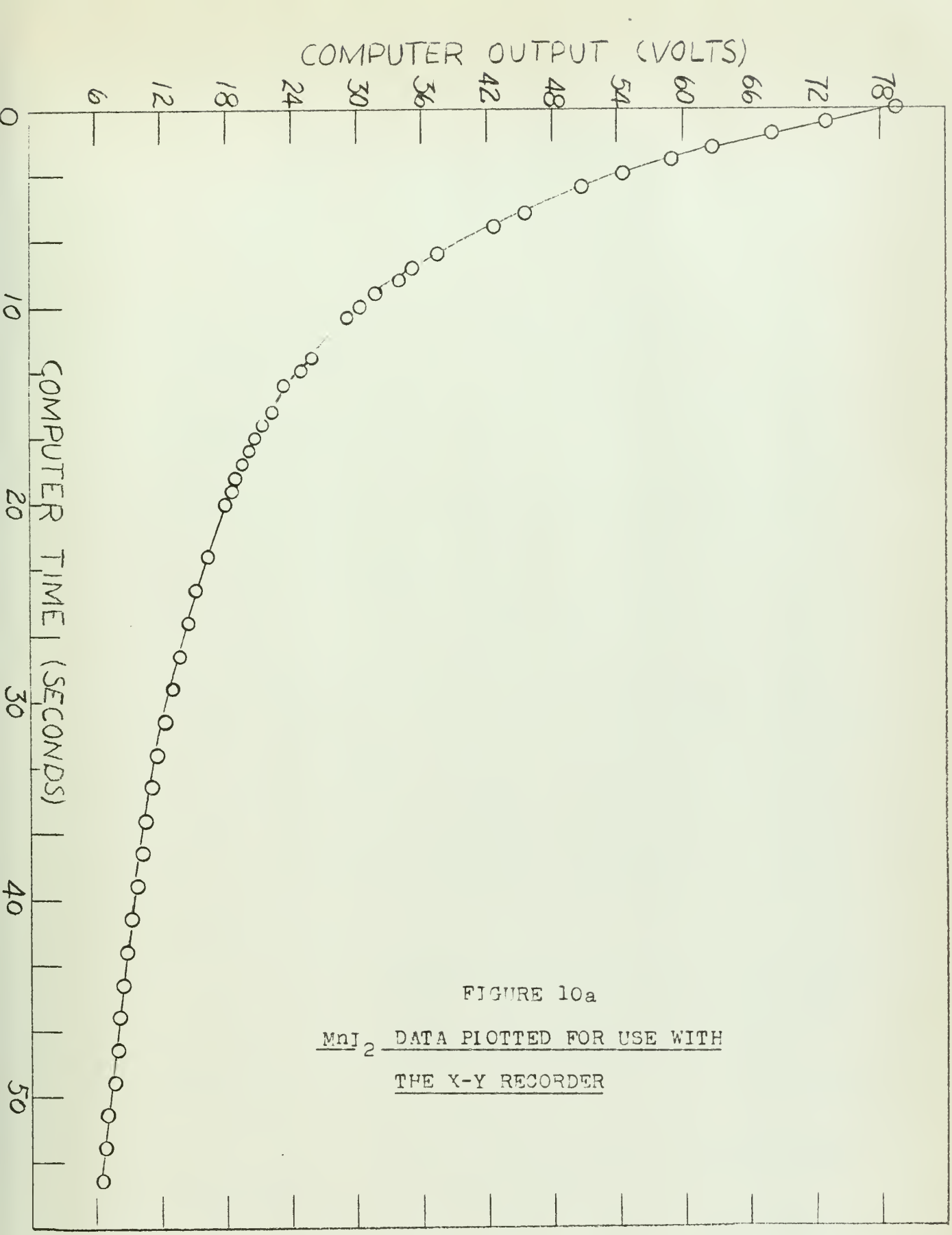


FIGURE 10a

MnI_2 DATA PLOTTED FOR USE WITH THE X-Y RECORDER

FIGURE 10b

SCALING FACTORS AND RESULTS

(MnI_2 DATA)

SCALED ACTIVITY $a = \frac{\text{ACTUAL ACTIVITY}}{\alpha}$				
COMPUTER TIME $\tau = \beta t$ ACTUAL TIME COEFFICIENT POTENTIOMETER $K = \Phi\lambda$ ACTUAL SLOPE				
INITIAL VALUE VOLTAGE $\bar{A}_0 = \gamma a$ SCALED ACTIVITY				
$\alpha = 300$	$\beta = \frac{1}{10}$	$\Phi_1 = 1$ $\Phi_2 = \frac{1}{10}$	$\gamma_1 = 1$ $\gamma_2 = 1$	
$\bar{A}$	$\bar{A}_0 = 27.09 \times \frac{\alpha}{\gamma_1} = 8127$			Mn^{56}
	$K_1 = .2690 \times \frac{1}{\Phi_1} = .2690$			
$\bar{B}$	$\bar{B}_0 = 51.85 \times \frac{\alpha}{\gamma_2} = 15555$			I^{128}
	$K_2 = .1664 \times \frac{1}{\Phi_2} = 1.664$			

SAMPLE #1

Mn^{56} relative activity at t=0: 8127 counts
slope .2690 hours^{-1}, $t_{\frac{1}{2}}$ = 2.58 hours

I^{128} relative activity: 1555 counts
slope 1.664 hours^{-1}, $t_{\frac{1}{2}}$ = 25.0 minutes

SAMPLE #2

Mn^{56} relative activity : 4600 counts
slope .2690 hours^{-1}, $t_{\frac{1}{2}}$ = 2.58 hours

I^{128} relative activity : 8630 counts
slope 1.666 hours^{-1}, $t_{\frac{1}{2}}$ = 25.0 minutes

FIGURE 10c

COMPUTER PROGRAM TO RESOLVE

THE MnI_2 DATA

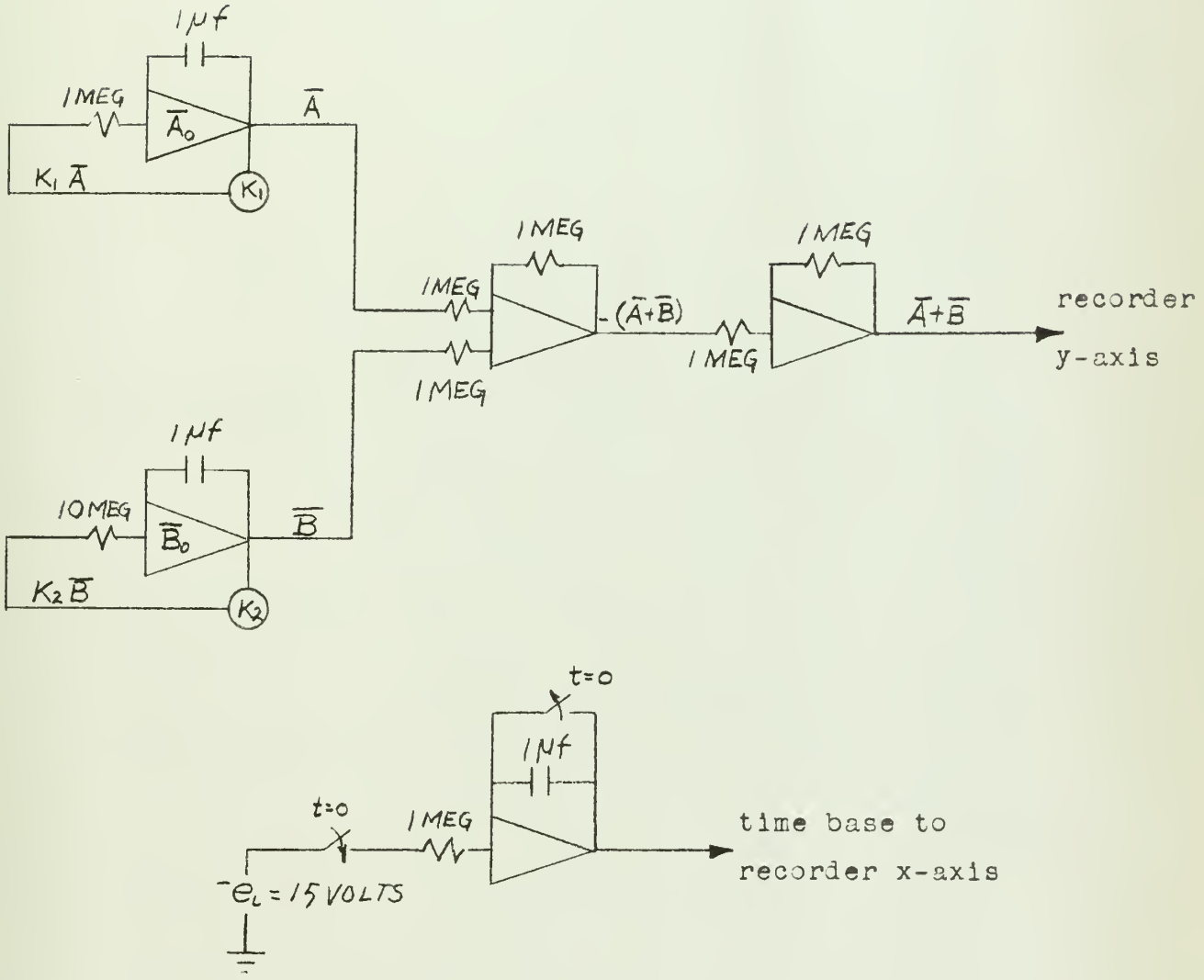
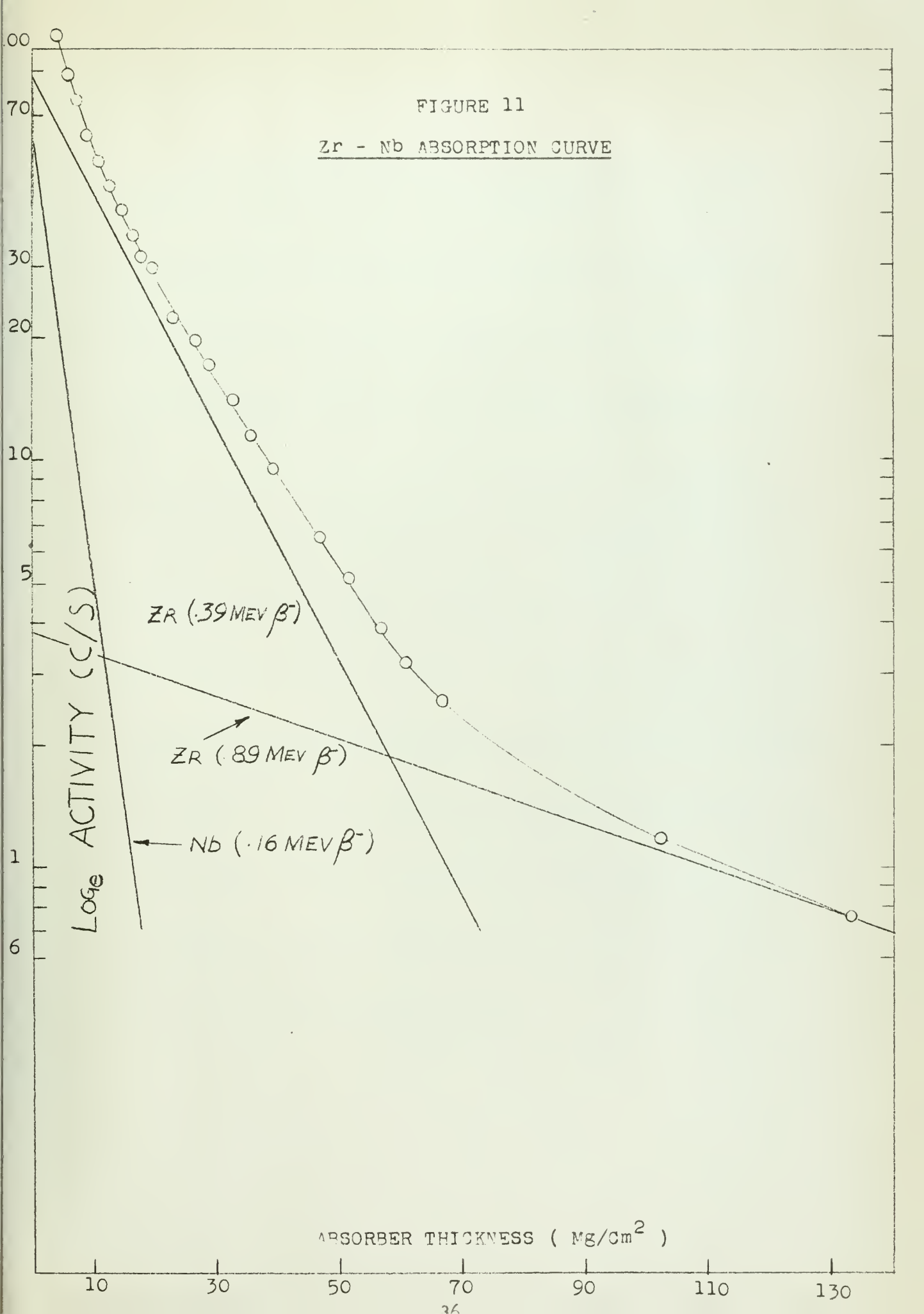


FIGURE 11

Zr - Nb ABSORPTION CURVE



BIBLIOGRAPHY

1. R.A.Lauderdale and A.H.Emmons, "Decontamination of Small Volumes of Radioactive Water", Nucleonics, 8, pp 21-26, May 1955.

2. R.A.Reinhardt and G.F.Kinney, "Operational Analogues for Kinetic Studies", USNPGS Research Paper No.15, November 1957.

3. Operating Handbook Model 30 Analog Computer, Donner Scientific Co., Berkeley, California.

4. Operating Manual, Sanborn Dual Channel DC Amplifier Model 150-2000, Sanborn Co., Cambridge, Massachusetts, 1954.

5. Instruction and Operating Manual for Model 3 Autograph X-Y Recorder, F.I.Moseley Co., Pasadena, California, 1956.

6. Instruction and Operating Manual for Autograph Curve Follower, F.I.Moseley Co., Pasadena, California, 1956.

7. Operating Handbook Model 50 Comparator Bridge, Donner Scientific Co., Berkeley, California.

8. Instruction Manual for Scaler, 1070A, Atomic Instrument Co., 1954.

9. J.V.Morey, "Intercalibration Between a Vial Scintillation Detector and a Geiger-Mueller Counter", Bachelor of Science Thesis, Library USNPGS, 1955.

10. R.C.H.Wheeler, "Basic Theory of the Electronic Analog Computer", Donner Scientific Co., Berkeley, California, 1955.

11. G.A.Korn and T.M.Korn, "Electronic Analog Computers", 1st Ed., McGraw Hill Book Company, Inc., New York, 1952.

12. J.M.Hollander and G.T.Seaborg, "Table of Isotopes", Rev. Mod. Phys., 30, p 585, April 1958.

13. CHART OF THE NUCLIDES, 5th Ed., Knolls Atomic Power Laboratory, revised to April 1956.

14. H.E.Shaw, from unpublished laboratory data.